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A Pitot Tube Method for Measuring the First Normal Stress Difference and Its Influence on Laminar Velocity Profile Determinations

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A scheme is outlined which enables the calculation of the normal stress difference P_{11}-P_{22} in fluids undergoing steady Poiseuille flow from measurements with a pitot tube of local velocity distributions. This new technique should enable measurements in a region of shear rate comparable to that encountered in the torsional and circular flow methods. A particularly interesting consequence of the method is it leads one to the inescapable conclusion that the presence of finite normal stresses should produce significant aberrations on the measurement of the velocity profile in the laminar flow of viscoelastic fluids. The presence of a P_{11}-P_{22} induced aberration on the velocity profile measurement suggests that caution be exercised in the practice of deducing estimates of the thickness of a boundary layer in the nonlaminar flow of these fluids.

Considerable effort is being given by experimental rheologists to the problem of developing tools for the quantitative measurement of normal stresses in viscoelastic fluids. Although there does not appear to be a universally preferred technique, the various methods currently used can be sharply divided into two main categories: providing data primarily for the purpose of evaluating constitutive equations and measuring properties under those conditions which a fluid of interest will encounter in flow through actual process equipment. The torsional (*1* to *8*) and circular flow techniques (*1* to *4*, *9* to *11*) are typical of the first category. The jet expansion and thrust methods (*2*, *4*, *7*, *12* to *16*) illustrate the second category. With the application of appropriate assumptions, either method yields a reasonable description of the normal stresses as a function of shear rate. It is unfortunate, however, that the jet/thrust methods which have their origins in the viscometric flow familiar to rheologists as Poiseuille flow or tube viscometer flow are, for a variety of reasons, usually restricted to measurements of normal stresses at shear rates not less than 10^3 sec.$^{-1}$, whereas the torsional and circular flow methods seldom approach shear rates as high as 10^3 sec.$^{-1}$ Thus one is faced with the problem of extrapolating results over a decade or more of shear rate in order to compare results from different experimental techniques. Because the Poiseuille flow case is representative of the class of exactly solvable viscometer flows,* it would be desirable to be able to measure the normal stress under conditions of shear rate comparable to those encountered in the first category of methods.

The purpose of this paper is to outline a scheme for experimentally determining the first normal stress difference $(P_{11} - P_{22})$† at low shear rates from measurements of point values of dynamic pressures with a pitot tube under the steady laminar conditions of Poiseuille flow. A particularly interesting consequence of the proposed

* The torsional and circular flow methods represent a class of problems which are not compatible with the dynamical equations unless inertia is neglected.

† In the language of Coleman and Noll (*17*), the material function difference $[\sigma_2(\Gamma_R) - \sigma_1(\Gamma_R)]$.

method is that it leads one to the inescapable conclusion that the presence of finite normal stresses should produce *significant aberrations on the measurement of velocity distributions in the laminar flow of viscoelastic fluids.* Another interesting feature of the proposed method is that it invokes only those assumptions normally associated with Poiseuille flow.

THEORY

In Poiseuille flow, each point of the fluid moves, under the influence of a constant pressure gradient $\partial p/\partial z$, with constant speed along a straight line parallel to the generator of the cylinder. Here the surfaces of laminar shear are coaxial cylinders which undergo a translation parallel to the z coordinate, the velocity v_z in the z direction being a function only of radial position r. From the momentum equations, expressed in cylindrical polar coordinates, one obtains the following expressions for the variation of the stress deviator with radial position:*

$$\tau_{11}(r,z) = -p(0,z) + (P_{11} - P_{22})_r - \int_0^r (P_{22} - P_{33}) d\ln r \qquad (1a)$$

$$\tau_{22}(r,z) = -p(0,z) - \int_0^r (P_{22} - P_{33}) d\ln r \qquad (1b)$$

$$\tau_{33}(r,z) = -p(0,z) - (P_{22} - P_{33})_r - \int_0^r (P_{22} - P_{33}) d\ln r \qquad (1c)$$

Here $p(0,z)$ is the isotropic pressure at the center line of the tube. The only assumptions involved in the derivation of Equations (1) are that the fluid is incompressible and that steady state laminar conditions prevail throughout the fluid. This implies that aberrations arising from a developing velocity profile (*18, 19*) and memory type of entrance effects (*20*) are absent. One also assumes that the stresses are implicit functions of the rate of shear. It follows that P_w, the force per unit area measured at a pressure tap located in the wall of the tube, will be numerically equal to $-\tau_{22}(R,z)$, the radially directed stress exerted by the flowing fluid. Thus at a given point along the z coordinate, where steady state flow conditions exist, the experimentally determinable pressure P_w is seen to consist of the isotropic or static pressure $p(0)$ and a term arising from a normal stress difference, namely

$$P_w = -\tau_{22}(R) = p(0) + \int_0^R (P_{22} - P_{33})_r \, d\ln r \qquad (2a)$$

Similarly, it follows that P_i, the impact pressure registered when an incompressible viscoelastic fluid impinges on the opening of a pitot tube, oriented so that the opening is in the same plane as the wall tap and points upstream toward the approaching flow, should be numerically equal to $-\tau_{11}(r,z)$, the longitudinally directed stress exerted by the fluid. The experimentally determinable impact pressure P_i is seen from Equation (1*a*) to apparently consist of the isotropic or static pressure $p(0)$ and terms involving two normal stress differences. However, there is a third contribution to the measured impact pressure, the well-known inertial or dynamic pressure term $\frac{1}{2}\rho\overline{v_z(r)}^2$, and the total impact pressure becomes

$$P_i = -\tau_{11}(r) + \frac{1}{2}\rho\overline{v_z(r)}^2 \qquad (2b)$$

In arriving at Equation (2*b*) the upstream pressure on the stagnation streamline is taken to be the pressure which would exist at the point occupied by the probe if the latter were not present, and one assumes the undisturbed pressure is identical to the static pressure at the wall tap. One also requires that $\partial/\partial z$ of such quantities as τ_{21} and τ_{11} be zero, these equations applying to the tube flow when the flow is not disturbed by the presence of the pitot tube. Additionally, in using the momentum balance to arrive at this expression, surface tractions due to viscous shear have been neglected, as is customary for the Newtonian fluid. Such assumptions may or may not be any more crucial when applied to rheologically complex fluids. Their importance can, as in the Newtonian case, best be resolved by carefully conceived and conducted experiments. Since Equation (1*a*) also applies to the undisturbed flow, one may use it to eliminate $\tau_{11}(r)$ from Equation (2*b*) and obtain the result

$$= p(0) - (P_{11} - P_{22})_r + \int_0^r (P_{22} - P_{33})_r d\ln r + \frac{1}{2}\rho\overline{v_z(r)}^2 \qquad (2c)$$

Thus, if for the fluid of interest the equality of P_{22} and P_{33} is assumed,† and one connects the wall tap and the opening in the pitot tube to a differential pressure device, the resulting difference Δp at a given wall stress should be a direct measure of the sum of the dynamic pressure term and the value of the normal stress difference ($P_{11} - P_{22}$) evaluated at the radial position r, namely

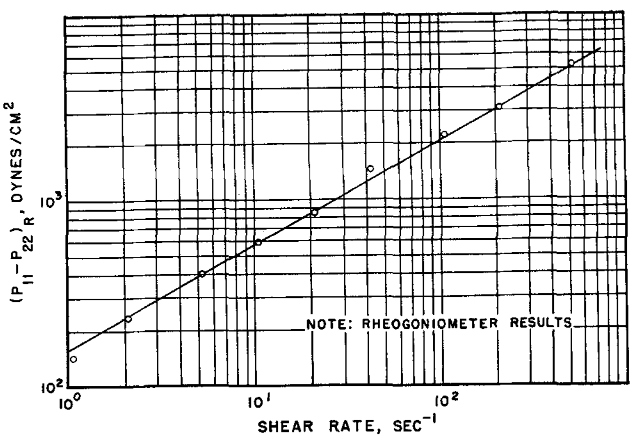

Fig. 2. First normal stress difference shear rate relationship 1% vinyl polymer in water.

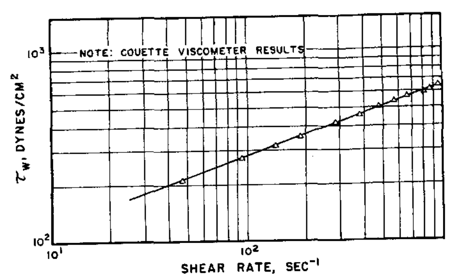

Fig. 1. Shear stress—shear rate relationship 1% vinyl polymer in water.

* The subscript notation 1, 2, 3 has been substituted for the axial z, radial r, and angular θ, directions, respectively.

† Recent experimental (*16*) data show that ($P_{22} - P_{33}$) is quite small compared with ($P_{11} - P_{22}$). If the same technique is applied to plane Poiseuille or channel flow, it is not necessary to invoke this assumption to obtain an identical result. The author is grateful to Professor A. B. Metzner for pointing this out.

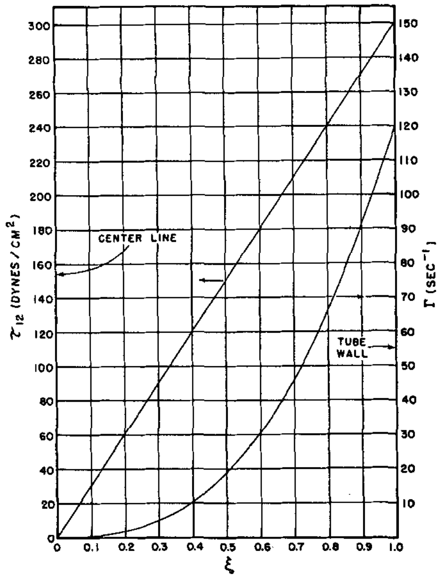

Fig. 3. Predicted radial variation of shear stress and shear rate for fluid exhibiting power law shear behavior.

$$\Delta p = P_\xi - P_w = -(P_{11} - P_{22})_r + \frac{1}{2}\rho\overline{v_z(r)}^2 \quad (3)$$

This sum can be evaluated over a range of shear rates up to a maximum Γ_R determined by the existing flow conditions simply by performing a conventional velocity transverse experiment. Note that the normal stress contribution vanishes on the center line where the rate of shear is zero, while the viscous contribution vanishes at the wall because of the no-slip hypothesis.

To extract the term $(P_{11} - P_{22})_r$ at a given radial position it is only necessary to know the value of the dynamic pressure at that point. It may be recalled that the velocity distribution in Poiseuille flow is given by (*21*)

$$v_z(r) = \frac{R}{\tau_w}\int_{\tau_{12}}^{\tau_w} f(\tau_{12})\,d\tau_{12} \quad (4)$$

where τ_w is the wall stress, and $\tau_{12} = \frac{r}{R}\tau_w$. Substituting for $v_z(r)$ from Equation (4) into Equation (3) one gets

$$(P_{11} - P_{22})_r = \Delta p - \frac{\rho R^2}{2\tau_w^2}\left[\int_{\frac{r}{R}\tau_w}^{\tau_w} f(\tau_{12})\,d\tau_{12}\right]^2 \quad (5)$$

In order to use Equation (5) directly to obtain $(P_{11} - P_{22})_r$ as a function of shear rate, one must measure the pressure difference Δp at a given radius and evaluate the term in brackets. As pointed out above, Δp is obtained from a measurement of the difference in pressures registered at the impact tube opening at radial position r and at the wall tap located in the same plane as the impact tube opening. The term in brackets can be readily evaluated numerically from tables of $f(\tau_{12})$ and τ_{12} obtained from a supplementary experiment involving measurements of wall stresses and flow rates under laminar flow conditions and use of the well-known formula (*21*)

$$f(\tau_w) = \frac{3Q}{\pi R^3} + \tau_w \frac{d(Q/\pi R^3)}{d\tau_w} \quad (6)$$

One may also obtain the required shear rate-shearing stress data from experiments in a well-designed Couette apparatus or from a plane Poiseuille or channel flow arrangement (*21*) using relations analogous to Equation (6). Alternatively, one may approximate the shearing behavior of the fluid of interest with an empirical formula such as the power law.

DISCUSSION

The possibility of an influence of the first normal stress difference $P_{11} - P_{22}$, of the type predicted by Equation (3), on a measurement of velocity distribution in the laminar flow of a viscoelastic fluid has not to the author's knowledge been previously discussed in the literature. It seems worthwhile therefore to predict the nature and magnitude of the aberration due to $(P_{11} - P_{22})$ which might be expected on a typical velocity profile measurement. For the purpose of simulating a velocity profile measurement, the author selected some normal* and shearing† stress data obtained on a 1% by weight solution of a vinyl polymer in water, the solution being markedly viscoelastic at this concentration. Figures 1 and 2 depict the $\tau_w - f(\tau_w)$ and $(P_{11} - P_{22}) - f(\tau_w)$ data, respectively. The viscometric data in Figure 1 can be approximated over the region of interest by the empirical power law

$$\left(\frac{-dv_z(r)}{dr}\right)_w = k\tau_w^N \quad (7)$$

The curve shown represents the least-squares curve as fitted with the aid of a numerical procedure. The power law constants derived from the fit are $N = 2.672$, $k = 2.89 \times 10^{-5}$ (dynes/sq.cm.)$^{-N}$sec.$^{-1}$ The density is taken as approximately unity. For the hypothetical velocity traverse experiment the following conditions were selected:

R = 2.403 cm.
τ_w = 300 dynes/sq.cm.
V = 51 cm./sec.
Γ_w = 120 sec.$^{-1}$

The Reynolds number corresponding is 69.36 which establishes that the flow is laminar. The following equations apply from the center line ($\xi = 0$) to the wall ($\xi = 1$)

* Obtained on a standard rheogoniometer.

† Obtained on an A37 Fann Couette type of viscometer.

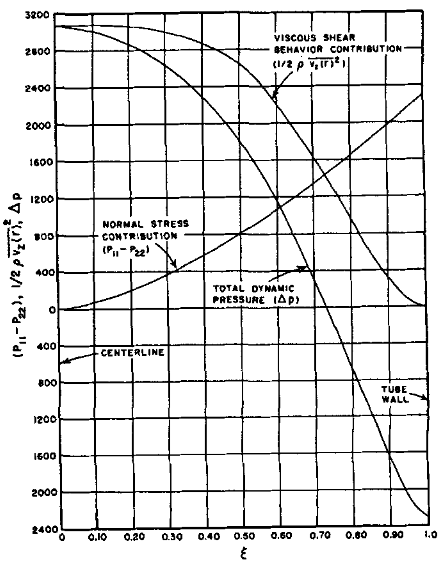

Fig. 4. Predicted contributions of viscous and normal stress properties to local dynamic pressures from pitot tube measurements.

for a fluid whose shear behavior is described by Equation (7):

$$\Gamma_r = \xi^N \Gamma_w \quad (8)$$

$$v_z(r) = \frac{R\,\Gamma_w}{N+1}\,(1-\xi^{N+1}) \quad (9)$$

$$\frac{1}{2}\,\rho\overline{v_z(r)^2} = \frac{\rho}{2}\left[\frac{R\,\Gamma_w}{N+1}\,(1-\xi^{N+1})\right]^2 \quad (10)$$

Local values of the shear stress and shear rate, computed from $\tau_{12} = \xi\tau_w$ and Equation (8), respectively, from the center line to the wall are shown in Figure 3. The variation of the dynamic pressure $\frac{1}{2}\rho\overline{v_z(r)^2}$ with radial position, calculated from Equation (10), is shown in Figure 4 as the curve labeled *viscous contribution.* When one notes the variation of shear rate with radial position from Figure 3, corresponding values of $(P_{11} - P_{22})_r$ are obtained by interpolation from the graph of $P_{11} - P_{22}$ vs. Γ_R in Figure 2. These normal stress differences are also shown in Figure 4 for comparison as the curve labeled *normal stress contribution.* It is seen from Equation (3) that at a given ξ, the pressure difference Δp observed experimentally between the opening of the pitot tube and the wall tap is the algebraic sum of two pressure terms: the force per unit area arising from a finite normal stress term $(P_{11} - P_{22})$, and the dynamic pressure due to the viscous contribution. The predicted variation of the local dynamic pressure with radial position calculated from Equation (3) for a fluid exhibiting both power law shear behavior and normal stress properties is shown as the curve labeled *total dynamic pressure* in Figure 4. The effect of finite values of the first normal stress difference is seen to lead to a striking aberration in the pitot tube experiment. Thus for the example cited here, an observer with no prior knowledge of the normal stress properties of the fluid would from the measured difference in impact and static pressure obtain the values of the total dynamic pressure shown in the region $0 < \xi < 0.732$ using a differential pressure measuring device in the conventional arrangement, that is, with the high and low sides connected to the impact opening and to the wall tap, respectively. To obtain the results shown in the region $0.732 < \xi < 1.0$ would require the reverse arrangement, with the low and high sides connected to the impact opening and to the wall tap, respectively.

The solid curve shown in Figure 5 represents the laminar velocity distribution computed for purely viscous power law shear behavior from Equation (9) for the example cited here. It follows that apparent local velocities v_a may be computed from the relation

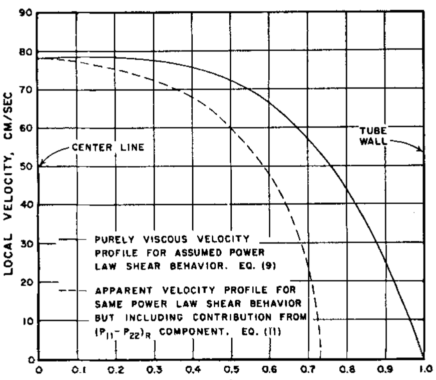

Fig. 5. Effect of (P_{11}-P_{22}) properties on the predicted velocity profile for a non-Newtonian fluid.

$$v_a = \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2}{\rho}\left\{\frac{1}{2}\,\rho\overline{v_z(r)^2} - (P_{11}-P_{22})_r\right\}} \quad (11)$$

that is, from the curve labeled *total dynamic pressure* in Figure 4 for the region $0 < \xi < 0.732$. The dashed curve shown in Figure 5 represents the apparent local velocities predicted from Equation (11) for the assumed flow conditions. These velocities may be interpreted as the velocities which an observer with no prior knowledge of the normal stress properties of the fluid would compute from the measured difference in impact and static pressure in the region $0 < \xi < 0.732$ using the expression $v_z(r) = \sqrt{\frac{2\Delta p}{\rho}}$. As a result of the $(P_{11} - P_{22})_r$ induced aberration on the measured local dynamic pressure Δp shown in Figure 4, the character of the laminar velocity distribution calculated from these measurements is now markedly different from the results one would predict from the purely viscous properties of the fluid. The presence of the first normal stress difference leads to a calculated profile which is considerably sharper than the blunt profile calculated from Equation (9). The effect is negligible only in the region around $\xi = 0$, where the shear rate is vanishingly small, becoming very pronounced as the wall is approached because the normal stresses increase with shear rate. Note that both pressure terms $(P_{11} - P_{22})_r$ and the contribution from the purely viscous behavior $\frac{1}{2}\rho\overline{v_z(r)^2}$ are of equal magnitude at $\xi = 0.732$. Thus from Equation (3), it follows that the dynamic pressure Δp would be zero at this radial position, with the result that the apparent local velocity computed from the measured pressure difference is also zero at $\xi = 0.732$ instead of at $\xi = 1.0$ as required by the no-slip hypothesis. On reaching this radial position, the observer would reverse the tap arrangement as outlined above to measure local dynamic pressures in the region $0.732 < \xi < 1.0$ for the cited example, otherwise negative apparent local velocities would be computed; for example, compare the curve labeled *total dynamic pressure* in Figure 4 with Equation (11). The local dynamic pressure measured at $\xi = 1.0$ is simply the value of the first normal stress difference evaluated at the maximum rate of shear. To extract $(P_{11} - P_{22})_r$ as a function of shear rate from these measurements, a suggested procedure is to construct a graph similar to Figure 4 and plot $\frac{1}{2}\rho\overline{v_z(r)^2}$, calculated for purely viscous behavior without the assumption of a functional relationship between shear rate and shear stress from the relation

$$\frac{1}{2}\,\rho\overline{v_z(r)^2} = \frac{\rho R^2}{2\tau_w^2}\left[\int_{\frac{r}{R}\tau_w}^{\tau_w} f(\tau_{12})\,d\tau_{12}\right]^2$$

or from Equation (10) if one uses the empirical power law, and values of Δp, the local dynamic pressures measured by the pitot tube technique, against radial position. One would use Equation (5) to obtain $(P_{11} - P_{22})_r$ as a function of radial position. Finally, Equation (8) provides the relatonship between radial position and shear rate if one adopts the power law, otherwise this would be provided by the tables of $f(\tau_{12})$, τ_{12}. It may become necessary to extrapolate to get $(P_{11} - P_{22})_R$, since it is well known that local velocity measurements near a boundary do not give accurate results because of the finite size of the impact tube opening. It is likely that the method will not be useful near the axis of the tube where the rate of shear is vanishingly small and the dynamic pressure term predominates. However, between these extremes the pitot tube method should be a useful technique for determining the variations of the normal stress difference with low rates of shear.

The current interest (*22*) in measuring velocity profiles in turbulent duct flow of viscoelastic fluids because of their drag reduction properties in this region is worth noting in connection with the effect of a $(P_{11} - P_{22})_r$ induced aberration on a profile measurement. That is, there is no reason to suspect that normal stress effects become inoperative outside the purely laminar flow region. Indeed, recent data by Metzner and Park (*23*) suggest that the observed drag reduction action is a function of the ratio of the elastic to viscous forces developed in the fluid. Thus, the thesis that the mechanism of drag reduction involves a change in thickness of the boundary layer, as deduced from experimentally determined profiles, uncorrected for the pressure contribution of the $(P_{11} - P_{22})_r$ term, requires reappraisal in the light of this previously unsuspected aberration. In this regard it is worth noting that aberrations in velocity profile measurements arising from finite normal stresses can be avoided by using hot wire or hot film anemometry techniques.

SUMMARY

A scheme is outlined which enables the calculation of the first normal stress difference $(P_{11} - P_{22})$ in fluids undergoing steady Poiseuille flow from measurements with a pitot tube of local velocity distributions. The development is free of any assumed relationships between the stresses and shear rate and as such is applicable to all fluids for which the equality of P_{22} and P_{33} can be assumed.

This pitot tube experimental technique enables measurements of $(P_{11} - P_{22})$ in a region of shear rate comparable to that encountered in the torsional and circular flow methods.

A particularly interesting result of this development is that it shows that the presence of this normal stress difference can lead to a very significant aberration in the local velocities which an observer would compute from the well-known expression $v(r) = \sqrt{\frac{2\Delta p}{\rho}}$ using measured Δp values. Although the interpretation of the velocity profile measurement is now more involved where the fluid of interest is rheologically complex because both the normal stress properties and the purely viscous properties contribute to the measured Δp, this analysis demonstrates that the $(P_{11} - P_{22})$ contribution can be readily separated from such measurements

The presence of a $(P_{11} - P_{22})$ induced aberration on the measurement of a velocity profile by the pitot tube technique suggests that caution be exercised, for the same reasons, in the practice of deducing estimates of the thickness of a boundary layer in the nonlaminar flow of rheologically complex fluids from velocity profile measurements under these flow conditions.

NOTATION

$f(\tau_w)$ = function defined by Equation (*6*)
k = parameter in power law defined by Equation (*7*)
N = parameter in power law defined by Equation (*7*)
$p(0, z)$ = isotropic pressure at center line of tube
P_{11}, P_{22}, P_{33} = components of the deviatoric normal stresses parallel to the streamlines, normal to the shearing surfaces, and normal to surfaces normal to the shearing surfaces and parallel to the streamlines, respectively
P_i = pressure measured by observer at opening in impact tube, defined by Equation (*2b*)
P_w = pressure measured by observer at wall tap in same plane as impact tube opening, defined by Equation (*2a*)
Q = volumetric flow rate
r = radial coordinate
R = tube radius
$v_z(r)$ = velocity component along z coordinate
V = average velocity
z = distance along axial coordinate

Greek Letters

Γ = local shear rate
Γ_w = shear rate evaluated at tube wall
ξ = reduced radius defined by $\xi = r/R$
ρ = fluid density
σ_1, σ_2 = material functions
$\tau_{11}, \tau_{22}, \tau_{33}$ = total normal stress components parallel to the streamlines, normal to the shearing surfaces, and normal to surfaces normal to the shearing surfaces and parallel to the streamlines, respectively
τ_{12} = local shear stress
τ_w = shear stress evaluated at tube wall

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